AN INVESTIGATION OF SNAP ROLL
IN SUBMARINES

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by

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ABSTRACT

Snap roll is a phenomenon which has plagued the sub-
marine designer since the advent of the high speed submarine.
A description of snap roll and its possible causes is pre-
sented in this thesis. A computer program is developed
which simulates a submarine in surge, sway, yaw and roll.
It is verified that snap roll is very sensitive to the
metacentric height. Additionally, it is seen that reducing
the sail size does not have as large an effect on reducing
the roll angle as anticipated. Rudder sequencing and speed
reduction are two other methods examined. The results of
the four investigations are presented in graphic form. The
conclusions include a justification for the use of the
simulation computer model as a design tool.

Thesis Supervisor: Martin A. Abkowitz
          Title: Professor of Ocean Engineering

ACKNOWLEDGEMENTS

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NOMENCLATURE

SYMBOL	DEFINITION
CB	Center of buoyancy.
CG	Center of gravity.
D	Drag force.
I_i	Moment of inertia about the i axis.
I_{jk}	Product of inertia about jk axis.
K	Hydrodynamic moment about x axis (rolling moment).
K'_i	Non-dimensional coefficient used in representing K as a function of i.
ℓ	Overall ship length.
L	Lift force.
m	Ship's mass.
N	Hydrodynamic moment component about z axis (yawing moment).
N'_i	Non-dimensional coefficient used in representing N as a function of i.
p	Angular velocity about the x axis.

$\dot{p}$	Angular acceleration about the x axis.
q	Angular velocity about the y axis.
$\dot{q}$	Angular acceleration about the y axis.
r	Angular velocity about the z axis.
$\dot{r}$	Angular acceleration about the z axis.
U	Velocity of the origin of the body axes relative to the fluid.
u	U velocity component in the x direction.
$\dot{u}$	Acceleration component in the x direction.
u_o	Initial steady value of U in the x direction.
v	U velocity component in the y direction.
$\dot{v}$	Acceleration component in the y direction.
w	U velocity component in the z direction.
$\dot{w}$	Acceleration component in the z direction.
x_B	The x coordinate of the center of buoyancy.
x_G	The x coordinate of the center of gravity.

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x_S The x coordinate of the center of pressure of the sail.

X Hydrodynamic force along x axis (axial force).

X'_i Non-dimensional coefficient used in representing X as a function of i.

y_B The y coordinate of the center of buoyancy.

y_G The y coordinate of the center of gravity.

y_S The y coordinate of the center of pressure of the sail.

Y Hydrodynamic force along y axis (lateral force).

Y'_i Non-dimensional coefficient used in representing Y as a function of i.

z_B The z coordinate of the center of buoyancy.

z_G The z coordinate of the center of gravity.

z_S The z coordinate of the center of pressure of the sail.

β Angle of drift.

δ_b Bowplane or sailplane deflection.

δ_r	Rudder deflection.
δ_s	Sternplane deflection.
η	Ratio u_o/u.
θ	Angle of pitch.
Υ	Angle of yaw.
φ	Angle of roll.
a_i, b_i, c_i	Constants used to represent the propeller thrust in the axial equation.

CHAPTER I - INTRODUCTION

I.1 BACKGROUND

Since the advent of the high speed submarine in the
late fifties, the submarine designer has been confronted
with new design problems in the area of ship control. One
of these problems, which is still with us two decades later,
is <u>snap</u> <u>roll</u>. Part of the reason for the perpetuation of
this problem has been a general lack of adequate data and
information concerning this phenomenon. Consequently,
appropriate design criteria could not be developed and the
problem of snap roll has continued. Naval architects and
ship designers need adequate alternatives and trade-offs to
provide good designs. This is the motivation for this
thesis.

A computer model will be developed in Chapter II which
simulates a submarine in roll, yaw, surge and sway. This
model can be used by naval architects as a design tool in
performing trade-off studies or as a method of establishing
design criteria for a particular design. The alternatives
for reducing snap roll, which are investigated in Chapter III
using this simulation model, were based on the immediate
needs of the design community and the time and monetary
constraints normally placed on this type of effort. It was
felt that these alternatives should not increase the com-

plexity of present designs or alter present design
practices, in order that they may be put to immediate use.
Chapter IV presents the conclusions and recommendations of
this thesis. In the appendix will be found a listing of
the computer model, a flow chart and a sample output.

I.2 WHAT IS SNAP ROLL?

An excellent description of snap roll was presented by
Griffin, et al (1)[1]:

"At 20 sec after full rudder, the initial
transients have died out, and the lateral
states begin to follow the trim values
which correspond to the decreasing forward
speed. Eventually, the forward velocity
reaches its trim value, and r, q, v, w
and φ are fixed at their respective trim
values. For r, q, v, and w, the difference
between the plateau value and the later trim
value is not large; however, for roll, it
can differ by more than 13 deg. This is
snap roll."

Simply stated, snap roll is the maximum roll amplitude
that results from a turning maneuver. The large roll angle,

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[1] Numbers in parentheses indicate references listed
at the end of this thesis

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which develops early in the turn, constitutes the "snap."
The large peak value of the roll angle is the submarine
tending towards its steady state turning value for the
initially high forward speed. This usually occurs about
20 seconds into the turn. As the submarine enters the
turn and during the initial phase of the turn, there is a
large side slip velocity. This side slip velocity probably
accounts for a major portion of the maximum roll angle.
The subsequent reduction of the large roll angle is pri-
marily due to speed loss. The high drag force developed
during the turn maneuver dramatically reduces the forward
velocity component. The steady state forward speed can be
expected to be 1/4 to 1/3 of the initial speed.

     There is still conjecture amongst hydrodynamicists as
to the theory and physics involved in explaining this
phenomenon. It is generally agreed that snap roll pri-
marily depends on the turning rate, the amount of rudder
deflection, the fairwater (sail) size and location, and the
ship speed. A large turning rate and/or rudder deflection
produces a large lateral velocity. Consequently, this large
v acting on the sail produces a large roll moment. Addi-
tionally, it has been proposed that the starting lift
exceeds the steady lift due to the dominating effect of the
trailing vortex sheet downwash in the steady case (2).
This may be due to the starting vortex formed and shed by

the sail. However, there is no substantiating evidence for
this.

CHAPTER II - THE SIMULATION MODEL AND COMPUTER PROGRAM

II.1 THE FOUR DEGREE OF FREEDOM MODEL

There exists at least three computer programs which simulate a submarine in a maneuver. However, none of these programs are available for general use, primarily due to proprietary considerations. Therefore, in order to investigate snap roll in submarines, it was necessary to develop another simulation model. Unlike the other models, however, this model involves the use of four degrees of freedom vice six degrees of freedom equations. The four coupled equations considered here are the X, Y, N and K equations. The justification for the use of only four equations of motion is that in a turn, depth excursions are small and the Z and M equations may be decoupled. The depth change which occurs during a turn, called "squat". normally can be easily remedied with a small deflection of the diving planes and/or sternplanes. These small excursions have a negligble effect on snap roll. This is evidenced by the fact that a four degree of freedom simulation program provides valid results for a turning maneuver. This will be seen from the results presented later in this chapter and in Chapter III. The advantages afforded by a four degree of freedom program, in addition to providing valid results, are:

- Reduced computation time.

- The estimation or measurement of fewer hydrodynamic coefficients.

Of course, such a model, necessarily restricts one's consideration to only maneuvers involving surge, sway, roll and yaw.

II.2   THE COORDINATE SYSTEM AND EQUATIONS

The coordinate system chosen for use in this model was based on reference (3). This reference gives the standard equations of motion for submarines used by the U.S. Navy. The coordinate system is illustrated in figure 2-1. It should be noted that the origin is centered at the center of gravity. A better choice may have been to use the center of buoyancy; however, to avoid a rewriting of already valid equations, the reference (3) system was used. The center of buoyancy of the basic hull for a submarine design is necessarily the geometric center of the hull. Therefore, a transverse velocity (or acceleration) would not produce a roll moment, caused by the basic hull form and this would reduce the number of necessary calculations.

The equations used in the development of this simulation model were taken directly from reference (3) and are listed in Appendix B. There are no second order or higher

Coordinate System

Figure 2-1

order acceleration terms involved in these equations. This is due to the lack of any significant interaction of the viscous and inertia forces. This allows the use of potential theory which gives adequate values for the hydrodynamic forces (4). Acceleration and velocity interaction terms are also assumed negligibly small for the same reason stated above. It should be noted that the equations remain non-linear even without these second order acceleration terms, due to the higher order velocity terms.

The equations are applied to a submarine in a turning maneuver under the following assumptions and restrictions:

1. The submarine is neutrally buoyant, i.e. $W - B = 0$.

2. The submarine is at least three hull diameters below the free surface, i.e. fully submerged.

3. The z component of the velocity is zero.

4. There is no initial list, i.e. $\varphi_0 = 0$.

5. There is zero trim, i.e. $\theta = q = \dot{q} = 0$.

6. The origin is at the center of gravity, i.e. $x_G = y_G = z_G = 0$.

7. Only the moments of inertia along the principal axes are non-zero, i.e. $I_{xy} = I_{yz} = I_{xz} = 0$. This is partly a consequence of #6.

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8. The coefficients and equations used are
   in non-dimensional form.

9. The control surface deflections are
   initially zero, with only the rudder
   angle being non-zero for t>0.

II.3 THE SOLUTION OF THE EQUATIONS OF MOTION

The simplest method of solving for the roll angle
induced by a turning maneuver is to solve for u(t) from
the <u>non-linear</u> X equation; then, to use this u in solving
the <u>linear</u> Y, N and K equations. This will provide com-
paratively accurate values for the roll angle for relatively
small deflections and for small forward speed losses. How-
ever, we are interested in high speed and tight turns,
where snap roll is most prominent. The vehicle motions in
this type of turning maneuver are outside the valid linear
range. Therefore, the non-linear equations of motion are
used.

The general method used for solving the four non-
linear equations is an iterative technique. The specific
technique employed in this thesis was adapted from the
method used in reference (5). The four equations are each
written in terms of the accelerations $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$. They
take the form:

$$X: \quad (m' - X'_{\dot{u}})\,\dot{u} - y_G\dot{r} = f_1\,(u,\, v,\, r,\, p,\, \delta_r)$$

$$Y: \quad (m' - Y'_{\dot{v}})\,\dot{v} - (m'z_G + Y'_{\dot{p}}\,1)\,\dot{p} +$$
$$(m'x_G - Y'_{\dot{r}}\,1) = f_2\,(u,\, v,\, r,\, p,\, \delta_r)$$

$$N: \quad (I'_z - N'_{\dot{r}})\,\dot{r} - (I'_{zx} + N'_{\dot{p}})\,\dot{p} +$$
$$(m'x_G/1^2 - N'_{\dot{v}}/1)\,\dot{v} - (m'y_G/1^2)\,\dot{u} =$$
$$f_3\,(u,\, v,\, r,\, p,\, \delta_r)$$

$$K: \quad (I'_x - K'_{\dot{p}})\,\dot{p} - (I'_{xz} + K'_{\dot{r}})\,\dot{r} -$$
$$(m'z_G/1^2 + K'_{\dot{v}}/1)\,\dot{v} = f_4\,(u,\, v,\, r,\, p,\, \delta_r).$$

The prime indicates a non-dimensional value.

Expressions for the accelerations $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$ are derived by employing matrix methods. Based on the initial conditions u_o, v_o, etc., starting values for $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$ are obtained for $t = 0$. Values for u, v, r and p are calculated from these starting values employing the Taylor expansions,

$$u(t + \Delta t) \simeq u(t) + \Delta t \cdot \dot{u}(t)$$
$$v(t + \Delta t) \simeq v(t) + \Delta t \cdot \dot{v}(t)$$
$$r(t + \Delta t) \simeq r(t) + \Delta t \cdot \dot{r}(t)$$
$$\varphi(t + \Delta t) \simeq \varphi(t) + \Delta t \cdot p(t) + ((\Delta t)^2) \cdot \dot{p}(t).$$

These new velocity values are then used to update the pre-

viously calculated accelerations. This indicates the
iterative nature of this integration technique. The value
of δ_r and t are updated after each iteration. The equations
are integrated over the time span specified. The size of Δt
is arbitrary; however, the smaller the Δt, the greater the
accuracy obtained from this technique. The accuracy desired
should be tempered by the fact that small Δt's require
rather large amounts of computer time. It was found that
Δt's on the order of one half to one second provided very
satisfactory results, while not using excessive amounts of
computer time.

II.4 COMPUTER SIMULATION MODEL VALIDITY

A large number of computer runs were executed for
various submarines, under various conditions. It was in-
tended to prove the validity of the developed computerized
simulation model using hydrodynamic coefficients taken from
towing tank model tests. The results from one of these
computer runs is presented in figure 2-2. As can be seen,
in general, the model accurately predicts the submarine's
response in roll; however, the predicted snap roll angle is
too large. This can be attributed to a number of causes.
One is the possibility of scale effects, since the co-
efficients used in the program are not the full scale
hydrodynamic coefficients. Secondly, the metacentric height
used was that specified in publications. The actual GM

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Full Scale Trials and Simulation Program

Full-scale trials

Simulation program w/model coefficients

Roll
Angle,
φ

Time, t

Figure 2-2

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during the full scale trials could vary slightly from this value and, as will be seen in Chapter III, the maximum roll angle is very sensitive to small changes in GM. However, for other submarines and sequences, the predicted maximum roll angles were less than the full scale values and therefore, this somewhat indicates that the difficulty in predicting the snap roll angle does not lie entirely in the model or technique used. It will also be noted that the model results increasingly deviate from the full-scale trials as t increases. This can be attributed to two causes. First, the full-scale trials are performed with a spiraling maneuver. In order to save computer time and since we were only interested in the maximum roll angle, the computer simulation runs were only performed for 180° turns. Therefore, the rudder angle was taken off earlier for the model and the roll angle decreased more rapidly for large t. Secondly, at slower speeds (large t), the cross coupling effect of pitch and heave on roll is more pronounced than at higher speeds. Since we are using only four degree of freedom equations, the effect of pitch and heave at low speeds is lost.

CHAPTER III - METHODS FOR REDUCING SNAPROLL

III.1   GENERAL
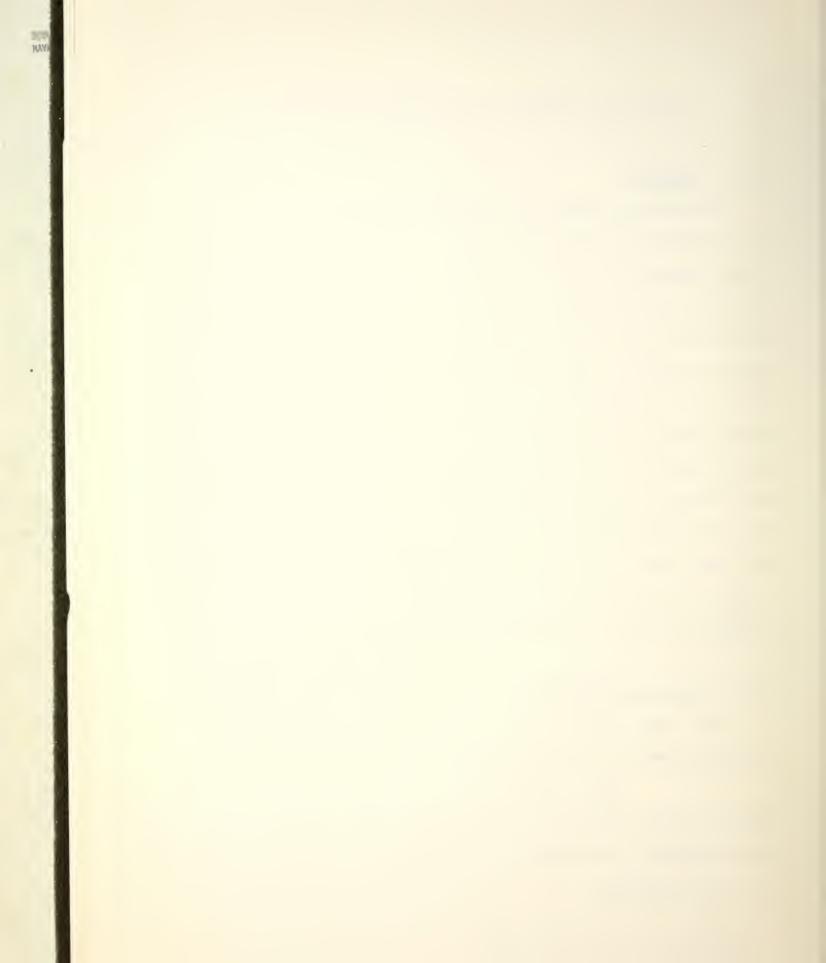
There are a number of techniques and methods which have
been proposed to reduce snap roll in submarines (1). Four of
these alternatives will be investigated here. As was in-
dicated in the introduction, the choice of these alter-
natives was based on the needs of the design community and
time constraints. The four methods chosen can be categorized
under one of two headings. The first concerns changes in the
naval architectural characteristics of the design. Increasing
GM and reducing sail size are classed under this category.
The second category involves those alternatives which make
use of an automatic ship control system. Rudder sequencing
and speed reduction fall under this classification. Each
method of snap roll reduction will be investigated indi-
vidually in the following sections.

III.2   METACENTRIC HEIGHT

The metacentric height is directly related to the
righting moment, K_φ, by the relation,

$$K_\varphi = -W \cdot GM$$

where W is the submerged displacement. This term is some-
what disguised in the governing equations in Appendix A.
In the K-equation,

$$K_\varphi = z_B \cdot B$$

where B is the buoyancy, and equal to the submerged displacement at neutral buoyancy, and z_B is the metacentric height. It can be shown that, as a first approximation, the maximum roll angle is inversely proportional to the metacentric height (1). Therefore, it would be advantageous to increase the metacentric height in order to reduce the effect of snap roll. However, this could prove to be a difficult design task, since it would involve adding weight or rearranging weights in an already weight-limited design.

The metacentric height for several submarines was increased by 0.25, 0.50, 0.75, 1.0 and 1.5 feet. A non-dimensional plot of the results are shown in figure 3-1. It should be noted that the amount of rudder used and u_o were different for each vehicle. This indicates that the percent decrease in the maximum roll angle is essentially independent of u_o and the maximum ordered rudder angle for a given GM change. The curve is asymptotic for greater than 100% changes in metacentric height. Therefore, increases in the metacentric height of greater than about 100% will yield no greater than approximately an 82% decrease in the maximum roll angle.

As an example, a 30% increase in GM yields a 33% decrease in φ_{max}. For a typical submarine, this translates into a 10° decrease in the maximum roll angle for an approx-

The Effect of GM on Snap Roll
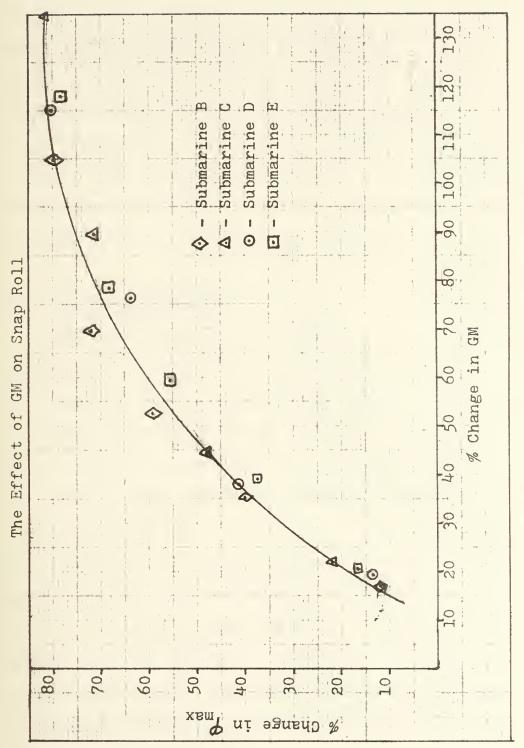
Figure 3-1

imately 4 inch increase in the metacentric height. It can be
seen that large decreases in the snap roll angle can be
realized for relatively small changes in GM. This is
evident in figure 3-2. This is a plot for a typical sub-
marine which shows the decrease in the maximum roll angle
for increasing GM. An average 5.75° decrease in φ_{max} is
gained for each 3 inch increase in GM. A point of minor
interest shown in this plot is that the maximum roll angle
occurs earlier in the turn as GM increases.

III.3 RUDDER SEQUENCING

    Rudder sequencing means to defer the full rudder angle
ordered, until some predetermined speed loss has been reached.
It is expected that by limiting the initial rudder angle,
the maximum roll angle will be reduced. This is based on
the fact that the roll angle is directly proportional to the
rudder deflection. Rudder sequencing is envisioned as part
of an automatic ship control system; however, the method of
rudder sequencing is simple enough to be employed as a
manual process. One disadvantage of this method of snap roll
reduction is that we might expect the transfer to increase.
However, such an increase is normally no more than one half
to one ship's length. The procedure employed in this thesis
to investigate this roll reduction method involves reducing
the maximum ordered rudder angle by 5, 10, 15 and 20 degrees.

Effect of Reducing GM on a Typical Submarine

Figure 3-2

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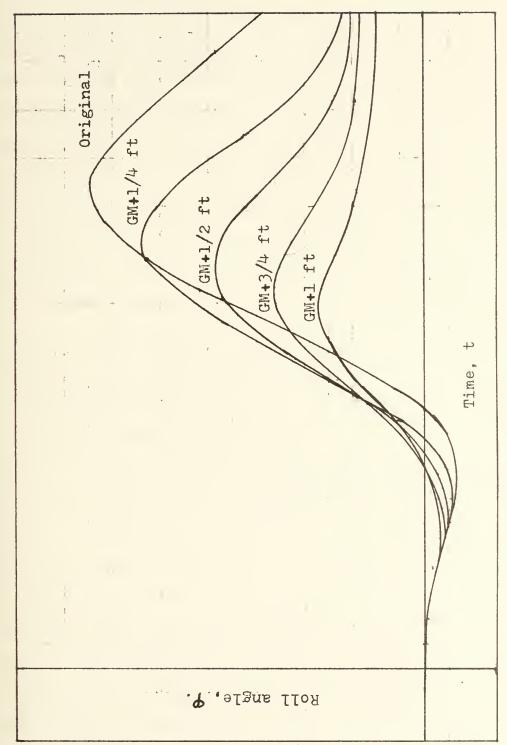
The predetermined speed loss criteria is to maintain the reduced rudder deflection until the speed in the turn has been reduced to one third that of the initial speed into the turn. This is depicted in figure 3-3. The selection of this speed criteria was based on the fact that the steady speed in a fully developed turn is approximately one third of the initial speed. The rudder deflection rate was held constant for all submarines and all sequences at 4 deg/sec.

The results are depicted in figure 3-4. As can be seen from the plot, this method of snap roll reduction is effective. However, relatively large rudder angle reductions are needed to produce small changes in φ_{max}. For example, a 10% reduction in the maximum roll angle requires a 23% reduction in the initial rudder angle. The use of rudder sequencing for a typical submarine is shown in figure 3-5. It can be seen that a greater decrease in φ_{max} per degree of rudder is gained for the 20° rudder sequence than for the 30° rudder sequence. It should be noted that some of the figure 3-4 data points were based on the same submarine at different initial speeds, i.e. 24 and 30 knots. Therefore, rudder sequencing seems to be independent of speed, in the sense that the same benefit is received for a particular rudder sequence despite the initial speed. For example, if a 20 degree rudder sequence reduces snap roll by 15% for u_o = 20 kts, then the same sequence will also reduce snap roll by 15% for a u_o = 30 kts.

Rudder Sequencing
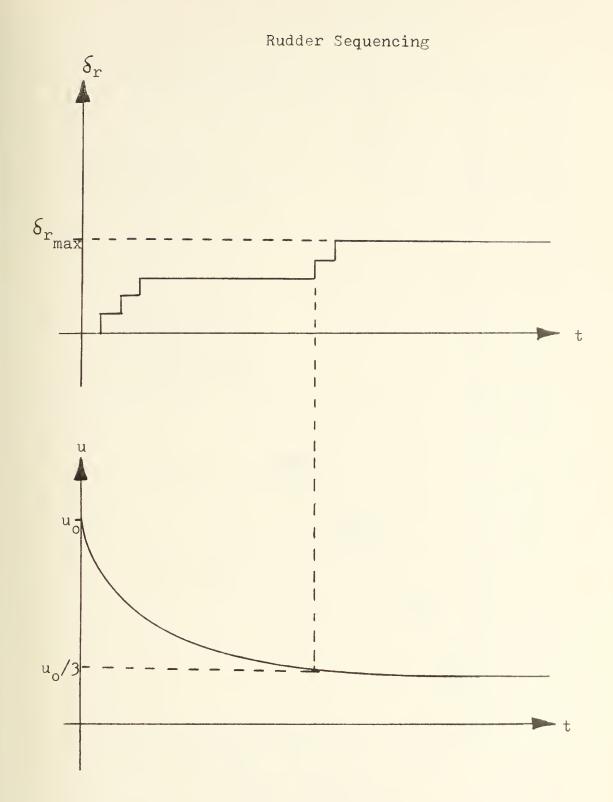
Figure 3-3

Effect of Rudder Sequencing on Snap Roll

□ Submarine B (u_o = 24 kts)
○ Submarine B' (u_o = 30 kts)
△ Submarine C (u_o = 25 kts)
◇ Submarine D (u_o = 20 kts)

$\dot{\delta}_r$ = constant = 4°/sec

% Change in (δ_r) initial

% Change in ϕ_{max}

Figure 3-4

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Rudder Sequence for a Typical Submarine

$(\delta_r)_{max} = 35$ deg

$\dot{\delta}_r = 4$ deg/sec

Original 35° rudder

30° rudder sequence

20° rudder sequence

Roll
Angle,
φ

Time, t
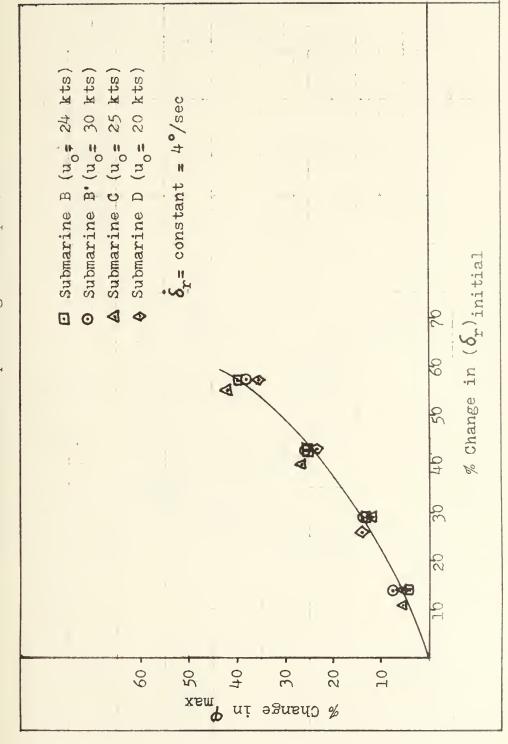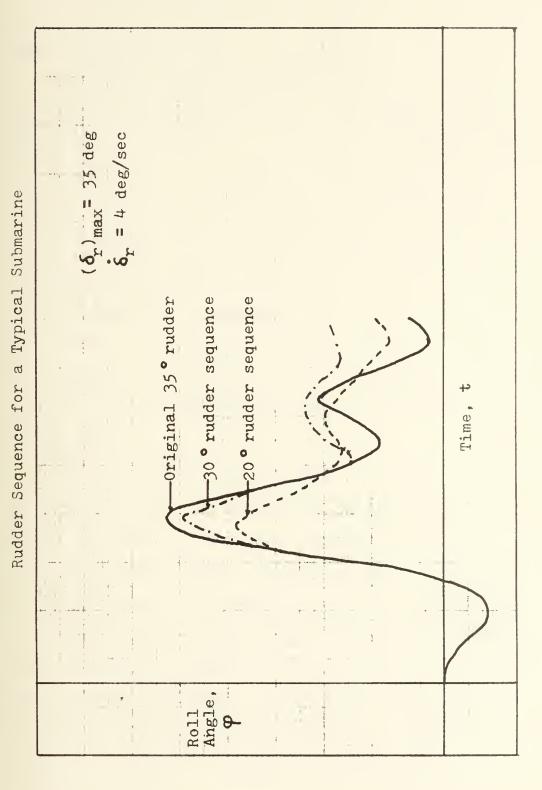
Figure 3-5

III.4   SAIL SIZE
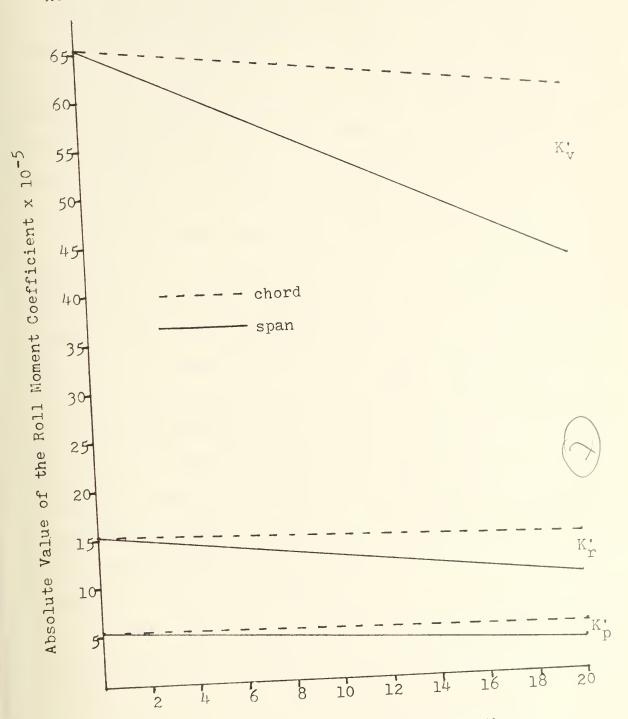
A submarine sail produces a large roll moment which is caused by the sideslip velocity. The magnitude of the roll moment varies widely from submarine to submarine, primarily due to the extensive variation in sail location and size. The sail dimensions are primarily derived from considerations for the quantity of equipment and the dimensions of the equipment to be placed in the sail. With the exception of the fairwater, which encloses the sail, the naval architect pays little attention to the hydrodynamic considerations of sail design. It was felt that if the sail size could be reduced, that a corresponding reduction in snap roll would be realized.

An investigation was performed to determine if the chord or the span had the greater impact on snap roll. Figure 3-6 shows the results. Obviously, a change in the span will have a greater impact on roll, as evidenced by the larger slopes for the three coefficients considered. This result was anticipated since the moment arm associated with the spanwise location of the center of pressure will have a greater influence on roll than the moment arm associated with the chordwise location. Additionally, a reduction in span will produce a corresponding reduction in

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Reduction in Roll Moment Due to Sail Size Reduction

Figure 3-6

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the lift due to the lower aspect ratio. A decrease in the
chord would obviously increase the aspect ratio. Conse-
quently, the span was reduced by 1, 5, 10, 15, 20 and 30
percent for a number of submarines. The results from these
computer runs will be presented; however, first the method
used to predict the impact on the hydrodynamic coefficients
of reducing the span will be presented.

The approximate method for determining the impact of
the sail on the hydrodynamic coefficients was partially
derived from Abkowitz's presentation for a lifting foil in
reference (4). This approximate method involves determining
the Y force, N moment and K moment at the sail as a result of
velocity disturbances v, r, and p. These coefficients are
reduced correspondingly for a reduction in the sail size.
The new sail coefficients are added to the original co-
efficients, less the original sail contribution. Then the
equations of motion are solved for φ. Figure 3-7 defines
the coordinate system, the variables involved in the
derivation, and sign conventions.

For most purposes, the sail x and y velocity components
are the same as the submarines's x and y velocity components,
i.e.

$$u = u_s \qquad v = v_s.$$

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The Coordinate System and Sign Conventions
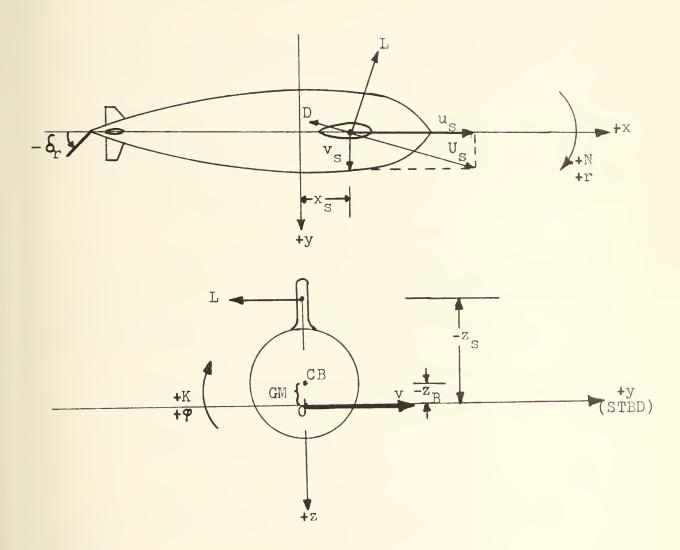
Figure 3-7

The subscript "s" denotes the local condition at the sail. The angle of attack produced at the sail by the transverse velocity is

$$\beta = \tan^{-1} \frac{v_s}{u_s} = \tan^{-1} \frac{v}{u} .$$

Consequently, we can calculate the Y force and N moment produced at the sail as

$$Y_s = - (L \cos\beta + D \sin\beta)$$

$$N_s = Y_s \cdot x_s$$

where L and D are the lift and drag forces on the sail, respectively. In terms of their respective non-dimensional coefficients, the lift and drag are

$$L = C_L \cdot \tfrac{1}{2} A_s U_s^2 = C_L \cdot \tfrac{1}{2}\rho A_s (u_s^2 + v_s^2)$$

$$D = C_D \cdot \tfrac{1}{2}\rho A_s (u_s^2 + v_s^2)$$

where A_s is the projected sail area and ρ is the fluid density.

From airfoil theory (6), the slope of the lift coefficient curve versus the angle of attack is shown to be approximated by

$$\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1 + 2/AR} \cdot$$

AR is the "reflected aspect ratio." This is equal to twice the mean span divided by the mean chord, which corrects for the proper pressure distribution on the sail. Using the above relations, it can be shown that the Y force on the sail due to a lateral velocity, v, is

$$(Y_v)_S = -\tfrac{1}{2}\rho A_S u_S \left(\frac{2\pi}{1 + 2/AR} + C_D\right)$$

and the N moment due to v is

$$(N_v)_S = (Y_v)_S \cdot x_S \cdot$$

Similarly, for the K moment

$$(K_v)_S = (Y_v)_S \cdot z_S$$

For the effect of an angular velocity, v, where $v_S = x_S \cdot r$, we find

$$Y_S \text{ due to } r = (Y_v)_S \cdot x_S \cdot r$$

$$N_S \text{ due to } r = (Y_v)_S \cdot x_S^2 \cdot r \qquad \text{and}$$

$$K_S \text{ due to } r = (Y_v)_S \cdot x_S \cdot z_S \cdot r \quad \cdot$$

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Taking the derivative of these coefficients with respect to
r,

$$(Y_r)_S = (Y_v)_S \cdot x_S$$

$$(N_r)_S = (Y_v)_S \cdot x_S^2$$

$$(K_r)_S = (Y_v)_S \cdot x_S \cdot z_S \quad .$$

A similar process for the contribution of an angular velocity
with $v_S = -z_S \cdot p$, yields

$$(Y_p)_S = -(Y_v)_S \cdot z_S$$

$$(N_p)_S = -(Y_v)_S \cdot x_S \cdot z_S$$

$$(K_p)_S = -(Y_v)_S \cdot z_S^2 \quad .$$

A similar derivation provides the hydrodynamic coefficients
for the sail with respect to the accelerations $\dot{v}$, $\dot{r}$ and $\dot{p}$.
The key coefficient is, of course, $(Y_{\dot{v}})_S$. Based on the added
mass calculation for a flat plate of dimensions $s/2$ and c,
$(Y_{\dot{v}})_S$ is

$$(Y_{\dot{v}})_S = -\frac{1}{2}\left[\frac{\pi \rho_S^2 c^2}{4\sqrt{s^2 + c^2}} \right] \quad .$$

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This form of $(Y_{\dot{v}})_S$ is used with a correction factor developed by Abkowitz (1977) and finally becomes

$$(Y_{\dot{v}})_S = -\tfrac{1}{2} \left[\frac{\pi \rho \bar{s}^2 \, \bar{c}^2}{4\sqrt{\bar{s}^2 + \bar{c}^2}} \right] \left[1 - \frac{0.54}{(1 + \bar{s}/\bar{c} + \bar{c}/\bar{s})} \right]$$

where $\bar{s}$ is the mean "reflected span" and $\bar{c}$ is the mean chord. The mean "reflected span" is directly related to the "reflected aspect ratio", in that, it corrects for the proper pressure distribution on the sail.

Since x_S and z_S are the distances from the center of pressure to the center of gravity along their respective axes, the following relations (7) proved beneficial,

$$(CP)_{\bar{c}} = 0.25 \cdot \bar{c}$$

$$(CP)_{\bar{s}} = (4/3)\pi \cdot \bar{s} \quad .$$

$(CP)_{\bar{c}}$ is the distance to the center of pressure chordwise location from the sail's leading edge. Similarly, $(CP)_{\bar{s}}$ is the distance from the root chord to the center of pressure.

Employing the above described procedure and equations, new coefficients were developed for reduction in sail size for a number of submarines. The results presented in figure 3-8 are for two different submarines, with what are

Change in the Snap Roll Angle due to a Reduction in Span
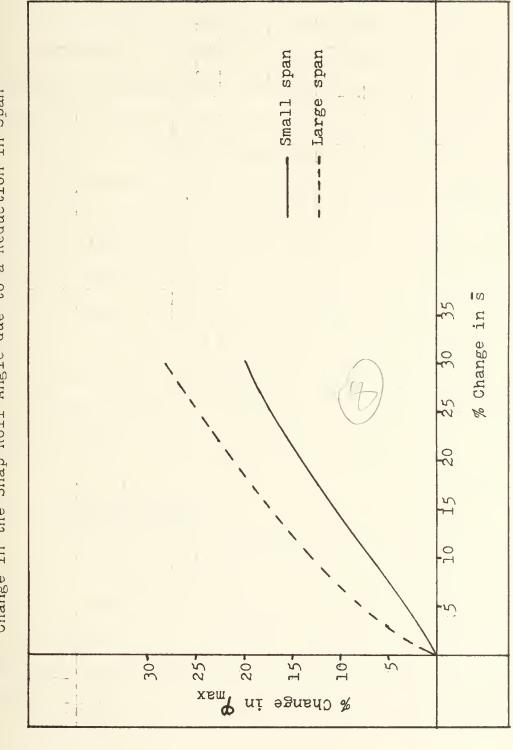
Figure 3-8

termed large and small spans. "Large" and "small" relate to
the relative size of the sails when compared to similar
submarine designs. It can be seen that a greater benefit
is realized by the larger sail. The maximum reduction
applied to the span was restricted to 30%. Any larger
reduction in span would not be feasible due to physical
restrictions, such as minimum periscope height requirements.

The decrease in the maximum roll angle through sail
size reduction was not as large as was expected. It is
believed that this may be explained by the fact that the
sail contributes to the roll damping at the same time that
it is acting as a roll producer. Therefore, by reducing
the sail size, the roll damping is reduced, as well as the
roll production. Fortunately, as indicated by figure 3-8,
roll damping appears to decrease less rapidly than roll
production for a given sail size reduction.

III.5 SPEED REDUCTION

It has been well established that the forward velocity,
u, has a strong influence on the roll angle when in a turn.
It can be shown that the maximum roll angle is proportional
to the square of the velocity. Therefore, it is expected
that small speed reductions will give significant reductions
in φ_{max}. This is an important point, since if a submarine

is designed for a maximum speed in excess of 30 kts, then the operating community would like to be able to take full advantage of this speed in various maneuvers, as well as in a straight line motion. The greatest difficulty with snap roll occurs at a submarine's top speed. It would be attractive, from an operational point of view, to be able to enter a turn at almost maximum speed. With this in mind, several submarines were simulated to enter turns at from one to four knots below their maximum speed. The results are presented in figure 3-9. As was expected, small changes in the velocity produce relatively large reductions in snap roll. For example, a 10% reduction in u, decreases the maximum roll angle by almost 20%. This method of reducing the effect of snap roll is equally feasible for use in an automatic control system or as a manual method of operation.

Percent Change in the Snap Roll Angle for Small Reductions in u

△ – Submarine C
⊙ – Submarine D
◇ – Submarine E

% Change in u

% Change in ϕ_{max}

Figure 3-9

CHAPTER IV - CONCLUSIONS AND RECOMMENDATIONS
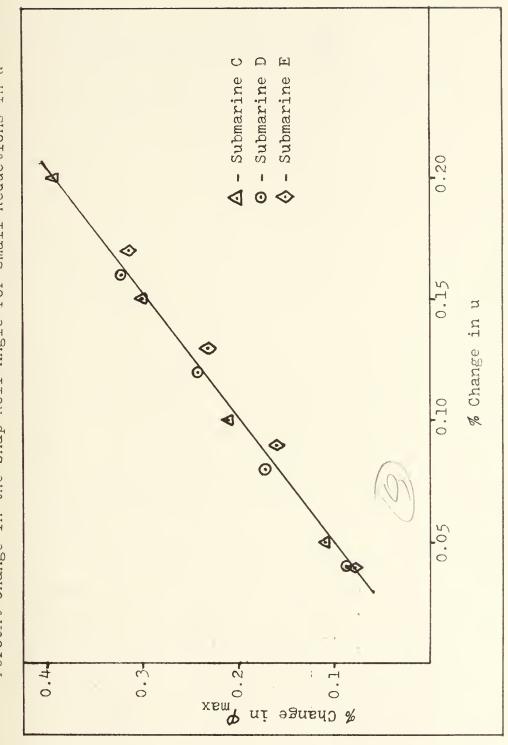
IV.1 CONCLUSIONS AND RECOMMENDATIONS

From this study, it must be concluded that we can, with
sufficient engineering accuracy, predict full-scale sub-
marine dynamic performance, at least in roll, from model
data. This is accomplished through the use of a computer
simulation model. This is of importance to the submarine
designer for three reasons. First, this prediction tech-
nique provides the ship designer with an invaluable design
tool. The stability characteristics of a multi-million
dollar submarine design can be accurately predicted prior to
making a major commitment to the procurement effort. The
ship designer now can proceed with confidence on a design,
at least from a dynamic stability standpoint. Secondly, the
computer model can be used by the submarine designer to
perturbate and iterate on a particular baseline design.
Consequently, such a technique could be employed to optimize
a design with respect to its stability criteria. Finally,
this method of predicting stability performance can be
employed by the designer to establish design criteria.
Without a foreknowledge of a submarine's performance in a
particular maneuver, the ship designer can test past
similar designs to establish some pattern of performance
and thereby derive a stability performance criteria.

It is apparent from the results of Chapter III that a small additional amount of GM in any submarine design will significantly reduce snap roll. Bearing this in mind, it would seem appropriate to suggest that the U.S. Navy revise its present design criteria for GM. The present criteria requires a minimum metacentric height, which does not work well for all designs from a stability standpoint. It is proposed that the revised design criteria for GM be based on a maximum acceptable snap roll angle. This would be a more flexible design criteria which could be tailored to individual designs. More importantly, it would couple a performance measure to the naval architectural characteristics. It would also accomodate for the variations in size of the roll moment producers which are encountered from design to design, i.e. the larger the roll moment producers, the larger the required GM to minimize the effect of snap roll.

The results of the other methods considered in Chapter III suggest obvious measures to be taken in an effort to reduce snap roll; these are to reduce speed, reduce initial rudder angle and reduce sail size. As was indicated in Section III.4, the reduction of sail size was not as effective in reducing snap roll as was anticipated. The possible reason for this was also suggested in Section III.4. However, a more thorough study in this area is certainly warranted. This is particularly true, in view of the fact

that an _approximate_ method was used for predicting the
appropriate hydrodynamic coefficients for the contribution
of the sail to snap roll. A more precise method of pre-
diction, coupled with a careful consideration of the total
effect of the sail and sailplanes on roll stability, could
lead to a better insight into the causes and minimization
of snap roll. Other recommendations for future investi-
gations are:

- The use of differential sailplanes and/or
  sternplanes as roll control fins.
- The use of unbalanced rudders.
- The effect of increasing the automation of
  submarine control systems on stability
  characteristics.

Three areas of improvement for the computer simulation
model are proposed. The first is the extension of the
present model to use the six degrees of freedom equations of
motion. This should not only improve the accuracy of the
results obtained from the present four degree of freedom
program, but it would also allow for the investigation of
more complex maneuvers and dynamic reactions, e.g. squat
with trim plus roll. Secondly, the model should be modified
to permit for more ease in its use by submarine designers.
The less complex the input requirements and the more lucid

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the output, the wider use the computer simulation model will experience within the design community. Lastly, it is recommended that the present program and future versions be optimized with respect to computer time. With the increasing expense and the time-sharing requirements of computer facilities, wider use of this type of computer simulation model will be realized by an efficient use of available computer time.

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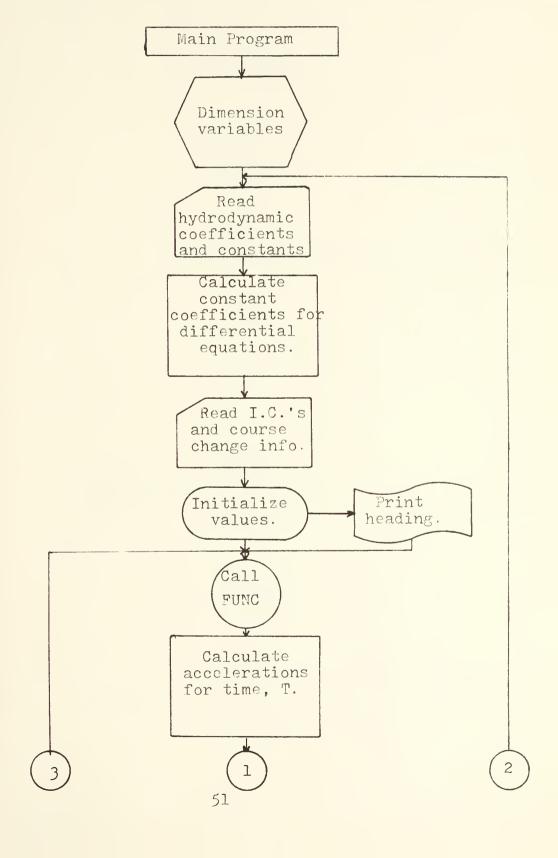
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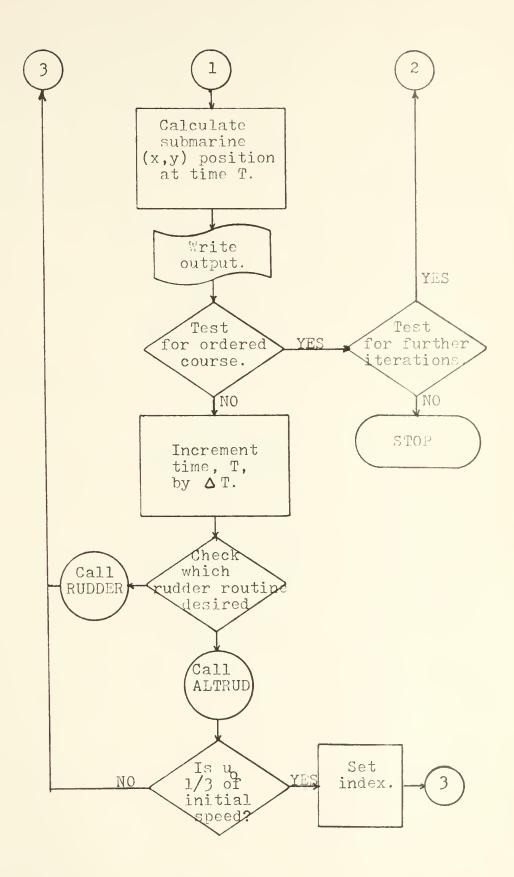
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<u>Language</u>, GC28-6515-10, May 1974.

A.1 FLOW CHART

```
                    ┌─────────────────────┐
                    │    Main Program     │
                    └─────────────────────┘
                               │
                               ▼
                         ⬡ Dimension
                           variables ⬡
                               │
                               ▼
                         Read
                      hydrodynamic
                      coefficients
                      and constants
                               │
                    ┌─────────────────────┐
                    │     Calculate       │
                    │     constant        │
                    │  coefficients for   │
                    │    differential     │
                    │     equations.      │
                    └─────────────────────┘
                               │
                         Read I.C.'s
                         and course
                         change info.
                               │
                               ▼
                    ( Initialize )  ──►   Print
                    (  values.   )       heading.
                               │
                               ▼
                          ( Call )
                          ( FUNC )
                               │
                    ┌─────────────────────┐
                    │    Calculate        │
                    │   accelerations     │
                    │   for time, T.      │
                    └─────────────────────┘
                               │
     ( 3 )                   ( 1 )                   ( 2 )
```

```
            ┌─────────────────────┐
            │     Subroutine      │
            │       FUNC          │
            └─────────────────────┘
                      │
                      ▼
                 ╱──────────╲
               ╱  Initialize  ╲
              │      and        │
              │ dimensionalize  │
               ╲  variables.   ╱
                 ╲──────────╱
                      │
                      ▼
                 ╱────────╲
               ╱    Test    ╲
              ╱ for appropriate╲
              ╲  thrust/drag  ╱
               ╲   coeff.    ╱
                 ╲────────╱
                      │
                      ▼
            ┌─────────────────────┐
            │     Calculate       │
            │  right-hand-side    │
            │  of differential    │
            │    equations.       │
            └─────────────────────┘
                      │
                      ▼
                ╭──────────╮
               │   Return   │
               │     to     │
                ╲   MAIN.   ╱
                ╰──────────╯
```

```
          ┌─────────────────────┐
          │     Subroutine      │
          │      RUDDER         │
          └──────────┬──────────┘
                     │
              ┌──────▼──────┐
             ╱  Initialize   ╲
            ╱       and       ╲
           ╱  dimensionalize   ╲
            ╲   variables.     ╱
             ╲────────┬───────╱
                      │
                 ╱────▼────╲
                ╱   Test    ╲
               ╱ where vehicle╲
               ╲  is in the   ╱
                ╲    turn.    ╱
                 ╲────┬────╱
                      │
          ┌───────────▼──────────┐
          │    Increment or      │
          │     decrement        │
          │     rudder as        │
          │    appropriate.      │
          └───────────┬──────────┘
                      │
               ╭──────▼──────╮
               │  Return to  │
               │    MAIN.    │
               ╰─────────────╯
```

```
        ┌─────────────────────┐
        │     Subroutine      │
        │      ALTRUD         │
        └─────────────────────┘
                  │
                  ▼
          ⬡ Initialize ⬡
          │    and      │
          │ dimensionalize │
          │  variables.  │
                  │
                  ▼
        ┌─────────────────────┐
        │    Adjust time      │
        │ increments due      │
        │   to delay in       │
        │    ordered          │
        │    rudder.          │
        └─────────────────────┘
                  │
                  ▼
              ◇ Test ◇
          ◇ where vehicle ◇
          ◇  is in the    ◇
              ◇ turn. ◇
                  │
                  ▼
              ◇ Test ◇
          ◇ when to ◇
          ◇ apply full ◇
              ◇ rudder. ◇
                  │
                  ▼
        ┌─────────────────────┐
        │    Increment        │
        │ or decrement        │
        │ rudder based        │
        │   on tests.         │
        └─────────────────────┘
                  │
                  ▼
          (  Return to   )
          (    MAIN.     )
```

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```
C      MAIN PROGRAM --- READ DATA & INITIAL CONDITIONS & INITIAL VALUES.      MAIN0001
C                       INITIALIZE & NON-DIMENSIONALIZE.  CALCULATE LINEAR D.E. MAIN0002
C                       COEFFICIENTS, ACCELERATIONS, VELOCITIES AND WRITE OUTPUT. MAIN0003
C                                                                              MAIN0004
C                                                                              MAIN0005
       IMPLICIT REAL*8(A-H),REAL*8(J-Z)                                        MAIN0006
       REAL*8   YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(       MAIN0007
      13),BI(3),CI(3),XCOEFF(16)                                              MAIN0008
       REAL*8 IX,IY,IZ,IXY,IXZ,IYZ                                            MAIN0009
       REAL ID(40)                                                            MAIN0010
       COMMON   T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG                         MAIN0011
       COMMON   XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG,       MAIN0012
      1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,EELB,THETA,PH     MAIN0013
      2I,A1,A2                                                                MAIN0014
       COMMON F1,F2,F3,F4,WT,B                                                MAIN0015
       COMMON DELR,RHO,TR,RUD1,U3                                             MAIN0016
       COMMON I1,I2                                                           MAIN0017
C                                                                              MAIN0018
C                                                                              MAIN0019
       READ NON-DIMENSIONAL COEFFICIENTS                                      MAIN0020
       INDEX=0                                                                MAIN0021
     1 READ(5,16,END=999) INDEX                                               MAIN0022
    16 FORMAT (I2)                                                            MAIN0023
       IF(INDEX) 2,2,3                                                        MAIN0024
     3 READ(5,10,END=999) (XCCEFF(I),I=1,16)                                  MAIN0025
       READ(5,10) (YCCEFF(I),I=1,22)                                          MAIN0026
       READ(5,10) (NCOEFF(I),I=1,22)                                          MAIN0027
       READ(5,10) (KCOEFF(I),I=1,17)                                          MAIN0028
       READ(5,10) (AI(I),I=1,3)                                               MAIN0029
       READ(5,10) (BI(I),I=1,3)                                               MAIN0030
       READ(5,10) (CI(I),I=1,3)                                               MAIN0031
    10 FORMAT(D10.4)                                                          MAIN0032
       READ(5,11) A1,A2                                                       MAIN0033
    11 FORMAT(2D10.4)                                                         MAIN0034
C      READ CONSTANTS -- MASS, LENGTH, ETC.                                   MAIN0035
       READ(5,12) XG,YG,ZG                                                    MAIN0036
       READ(5,12) XB,YB,ZB
```

```
      READ(5,12) M,L,RHO                                                MAIN0037
      READ(5,12) IX,IY,IZ                                               MAIN0038
      READ(5,12) IXY,IXZ,IYZ                                            MAIN0039
   12 FORMAT(3D10.4)                                                    MAIN0040
      READ(5,13) WT,B                                                   MAIN0041
   13 FORMAT(2D10.4)                                                    MAIN0042
C     CALCULATE COEFFICIENT VALUES FOR THE FOUR LINEAR D.E.'S           MAIN0043
      E(1)=0.0                                                          MAIN0044
      D(1)=0.0                                                          MAIN0045
      A(2)=0.0                                                          MAIN0046
      A(4)=0.0                                                          MAIN0047
      A(1)=M-XCOEFF(4)                                                  MAIN0048
      C(1)=YG                                                           MAIN0049
      E(2)=M-YCOEFF(6)                                                  MAIN0050
      D(2)=(M*ZG)+(YCOEFF(2)*L)                                         MAIN0051
      C(2)=(M*XG)-(YCOEFF(1)*L)                                         MAIN0052
      C(3)=IZ-NCOEFF(1)                                                 MAIN0053
      D(3)=IXZ+NCOEFF(2)                                                MAIN0054
      E(3)=(M*(XG/L**2))-NCOEFF(6)*(1/L)                                MAIN0055
      A(3)=M*(YG/L**2)                                                  MAIN0056
      E(4)=(M*(ZG/L**2))+KCOEFF(8)*(1/L)                                MAIN0057
      C(4)=IXZ-KCOEFF(2)                                                MAIN0058
      D(4)=IX-KCOEFF(1)                                                 MAIN0059
      DENOM=A(1)*(E(2)*C(3)*D(4)+C(2)*D(3)*E(4)+D(2)*E(3)*C(4)-E(4)*C(3) MAIN0060
     1*D(2)-C(4)*D(3)*E(2)-D(4)*E(3)*C(2))+C(1)*(D(2)*A(3)*E(4)-D(4)*A(3 MAIN0061
     2)*E(2))                                                           MAIN0062
C     READ INITIAL CONDITIONS                                           MAIN0063
    2 READ(5,14,END=999)    (ID(I),I=1,40)                              MAIN0064
   14 FORMAT(40A1)                                                      MAIN0065
      READ(5,15)  U,V,W,UDT,VDT,WDT                                     MAIN0066
      READ(5,15)  P,Q,R,PDT,QDT,RDT                                     MAIN0067
      READ(5,12)  THETA,PHI,PSI                                         MAIN0068
      READ(5,12)  DEIS,DELB,DELGM                                       MAIN0069
   15 FORMAT(6D10.4)                                                    MAIN0070
      ZB=ZB-DELGM                                                       MAIN0071
      UO=U                                                              MAIN0072
```

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```
      U=U*1.6889                                                        MAIN0073
      V=V*1.6889                                                        MAIN0074
      W=W*1.6889                                                        MAIN0075
      UDT=UDT*1.6889                                                    MAIN0076
      VDT=VDT*1.6889                                                    MAIN0077
      WDT=WDT*1.6889                                                    MAIN0078
C     READ COURSE CHANGE INFORMATICN                                    MAIN0079
      READ(5,15) TLAG,COURSE,RRATE,RUDAMT,RUD1,DELT                     MAIN0080
C     WRITE COURSE CHANGE INPUTS AND READINGS                           MAIN0081
      WRITE(6,49)    (ID(I),I=1,40)                                     MAIN0082
   49 FORMAT(1H1,39X,'*******************************************       MAIN0083
     1*',/,40X,'*',49X,'*',/,40X,'*',    '*',/,40X,'*',49X,'*',         MAIN0084
     2,/,40X,'*******************************************',',///)       MAIN0085
      WRITE(6,42)    UO,RRATE,COURSE,RUDAMT,DELT,RUD1                   MAIN0086
   42 FORMAT(33X,'UO =',D9.3,5X,'RUDDER RATE =',D9.3,5X,'COURSE CHANGE = MAIN0087
     1',D9.3,//,33X,'AMOUNT OF RUDDER =',D9.3,5X,'DELTA T =',D9.3,//,33X MAIN0088
     2,'RUD1 =',D9.3,////)                                              MAIN0089
      WRITE(6,41)                                                       MAIN0090
   41 FORMAT(5X,'T',9X,'U',9X,'V',9X,'R',8X,'PHI',7X,'UDT',7X,'VDT',7X,' MAIN0091
     1RDT',7X,'PDT',7X,'DELR',6X,'CRS',8X,'X',9X,'Y',//)                MAIN0092
      WRITE(6,44)                                                       MAIN0093
   44 FORMAT(3X,'(SEC)',5X,'(KTS)',5X,'(KTS)',5X,'(DEG/SEC)',3X,'(DEG)', MAIN0094
     13X,'(KTS/HRS)',1X,'(KTS/HRS)',1X,'(DEG/SEC)',1X,'(DEG/SEC)',3X,'(D MAIN0095
     2EG)',5X,'(DEG)',5X,'(YDS)',5X,'(YDS)',//,72X,'(SEC)',6X,'(SEC)',// MAIN0096
C     INITIALIZE TIME RUDDER QUANTITIES                                 MAIN0097
      T=0.0D1                                                           MAIN0098
      X=0.0D1                                                           MAIN0099
      Y=0.0D1                                                           MAIN0100
      I1=0                                                              MAIN0101
      I2=0                                                              MAIN0102
      TR=0.0D+1                                                         MAIN0103
      UO=UO*1.6889                                                      MAIN0104
C     CONVERT ANGLES IN DEGREES TO RADIANS FOR INTERNAL USE.            MAIN0105
      PHI=PHI/57.2958                                                   MAIN0106
      THETA=THETA/57.2958                                               MAIN0107
      PSI=PSI/57.2958                                                   MAIN0108
```

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```
      CCURSE=COURSE/57.2958                                             MAIN0109
      PRATE=RRATE/57.2958                                               MAIN0110
      RUDAMT=RUDAMT/57.2958                                             MAIN0111
      RUD1=RUD1/57.2958                                                 MAIN0112
      RUD=DABS(RUDAMT)                                                  MAIN0113
      T2=(RUD/RRATE)+TLAG                                               MAIN0114
      U3=U0/0.3D+1                                                      MAIN0115
      CRS1=DABS(COURSE)                                                 MAIN0116
      RTIME=CRS1/RRATE                                                  MAIN0117
      DELR=0.0D1                                                        MAIN0118
      IF(RTIME.LE.TLAG) GO TO 20                                        MAIN0119
      IF(T.EQ.0.0D1) GO TO 31                                           MAIN0120
C     CALL "FUNC" SUBROUTINE AND CALCULATE ACCELERATIONS AND VELOCITIES MAIN0121
C     AT TIME T + DELTA T.                                              MAIN0122
   30 PHI=PHI+(DELT*P)+((DELT**2)/2)*PDT                                MAIN0123
      PSI=PSI+(DELT*R)+((DELT**2)/2)*RDT                                MAIN0124
      U=U+DELT*UDT                                                      MAIN0125
      V=V+DELT*VDT                                                      MAIN0126
      R=R+DELT*RDT                                                      MAIN0127
      P=P+DELT*PDT                                                      MAIN0128
      CALL FUNC                                                         MAIN0129
C                                                                       MAIN0130
      UDT=(F1*(E(2)*C(3)*D(4)+C(2)*D(3)*E(4)+D(2)*E(3)*C(4)-E(4)*C(3)*D(MAIN0131
     12)-C(4)*D(3)*E(2)-D(4)*E(3)*C(2))+C(1)*(F2*E(3)*D(4)+E(2)*D(3)*F4+ MAIN0132
     2D(2)*F3*E(4)-F4*E(3)*D(2)-E(4)*D(3)*F2-D(4)*F3*E(2)))/DENOM        MAIN0133
C                                                                       MAIN0134
      VDT=(A(1)*(F2*C(3)*D(4)+C(2)*D(3)*F4+D(2)*F3*C(4)-F4*C(3)*D(2)-C(4 MAIN0135
     1)*D(3)*F2-D(4)*F3*C(2))-F1*(D(2)*A(3)*C(4)-D(4)*A(3)*C(2))+C(1)*(D MAIN0136
     2(2)*A(3)*F4-D(4)*A(3)*F2))/DENOM                                   MAIN0137
C                                                                       MAIN0138
      RDT=(A(1)*(E(2)*F3*D(4)+F2*D(3)*E(4)+D(2)*E(3)*F4-E(4)*F3*D(2)-F4* MAIN0139
     1D(3)*E(2)-D(4)*E(3)*F2)+F1*(D(2)*A(3)*E(4)-D(4)*A(3)*E(2)))/DENOM  MAIN0140
C                                                                       MAIN0141
      PDT=(A(1)*(E(2)*C(3)*F4+C(2)*F3*E(4)+F2*E(3)*C(4)-E(4)*C(3)*P2-C(4 MAIN0142
     1)*F3*E(2)-F4*E(3)*C(2))+C(1)*(F2*E(3)*A(3)-F4*A(3)*E(2))-F1*(C(2)* MAIN0143
     2A(3)*E(4)-C(4)*A(3)*E(2)))/DENOM                                   MAIN0144
```

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```
C
C     CALCULATE POSITION OF SUBMARINE (X,Y,)          AT TIME T.  ORIGIN TAKEN     MAIN0145
C     AT CG AT TIME = 0 .                                                          MAIN0146
      X=X+((V*DSIN(PSI)+U*DCOS(PSI))*DELT)                                         MAIN0147
      Y=Y+((U*DSIN(PSI)-V*DCOS(PSI))*DELT)                                         MAIN0148
C     WRITE OUTPUTS FOR TIME - T,                                                  MAIN0149
C     CONVERT ANGLES IN RADIANS TO DEGREES FOR OUTPUT.                             MAIN0150
   31 PDTO=PDT*57.2958                                                             MAIN0151
      RDTO=RDT*57.2958                                                             MAIN0152
      DELRO=DELR*57.2958                                                           MAIN0153
      PHIO=PHI*57.2958                                                             MAIN0154
      UCO=U/1.6889                                                                 MAIN0155
      VO=V/1.6889                                                                  MAIN0156
      PSIO=PSI*57.2958                                                             MAIN0157
      UDTO=UDT/1.6889                                                              MAIN0158
      VDTO=VDT/1.6889                                                              MAIN0159
      RC=R*57.2958                                                                 MAIN0160
      XX=X/3.0                                                                     MAIN0161
      YY=Y/3.0                                                                     MAIN0162
      WRITE(6,43) T,UOO,VO,RO,PHIO,UDTO,VDTO,RDTO,PDTO,DELRO,PSIO,XX,YY            MAIN0163
   43 FORMAT(13(1X,D9.3),/)                                                        MAIN0164
C                                                                                  MAIN0165
C     END CALCULATIONS WHEN ORDERED COURSE IS REACHED.                            MAIN0166
      CRS2=CRS1-DABS(PSI)                                                          MAIN0167
      IF(CRS2.LE.0.00435) GO TO 1                                                  MAIN0168
C                                                                                  MAIN0169
C     TIME INCREMENT ALGORITHM.                                                    MAIN0170
C                                                                                  MAIN0171
      IF(T.GE.TLAG) GO TO 21                                                       MAIN0172
      T=TLAG+DELT                                                                  MAIN0173
      GO TO 22                                                                     MAIN0174
   21 T=T+DELT                                                                     MAIN0175
C     UPDATE RUDDER ANGLE FOR TIME T.                                              MAIN0176
      IF(U.LE.U3) GO TO 23                                                         MAIN0177
   22 IF(RUD1.EQ.0.0D+1) GO TO 23                                                  MAIN0178
      CALL ALTRUD                                                                  MAIN0179
                                                                                   MAIN0180
```

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```
      GO TO 30                                                          MAIN0181
   23 CALL RUDDER                                                       MAIN0182
      GC TO 30                                                          MAIN0183
C     ERROR MESSAGE.                                                    MAIN0184
   20 WRITE(6,40)                                                       MAIN0185
   40 FORMAT(///,30X,'***    ERROR    ***',//,10X,'THE TIME LAG SPECIFIED MAIN0186
     1IS GREATER OR EQUAL TO THE TIME ALLOWED FOR THE TURN.  CHECK TLAG, MAIN0187
     2',/,10X,' RRATE AND COURSE.')                                     MAIN0188
  999 STOP                                                              MAIN0189
      END                                                               MAIN0190
```

```
C
      SUBROUTINE FUNC                                                   FUNC0001
C                                                                       FUNC0002
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)                                  FUNC0003
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(FUNC0004
     13),BI(3),CI(3),XCOEFF(16)                                         FUNC0005
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ                                       FUNC0006
      COMMON   T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG                    FUNC0007
      COMMON   XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,ODT,AI,BI,CI,XG,  FUNC0008
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,DELB,THETA,PH FUNC0009
     2I,A1,A2                                                           FUNC0010
      COMMON F1,F2,F3,F4,WT,B                                           FUNC0011
      COMMON DELR,RHC,TR,RUD1,U3                                        FUNC0012
      COMMON I1,I2                                                      FUNC0013
C                                                                       FUNC0014
C     SUBROUTINE FUNC CALCULATES THE THE RIGHT HAND SIDE OF THE FOUR NON-LINEAR FUNC0015
C     DIFFERENTIAL EQUATIONS.                                           FUNC0016
C                                                                       FUNC0017
      ETA=0.0D1                                                         FUNC0018
C     CALCULATE MAGNITUDE OF VECTOR VELOCITY.                           FUNC0019
      BIGU=DSQRT(U**2+V**2+W**2)                                        FUNC0020
      ETA=UO/BIGU                                                       FUNC0021
      THETAR=THETA                                                      FUNC0022
      PHIR=PHI                                                          FUNC0023
      VW=DSQRT(V**2+W**2)                                               FUNC0024
C     TEST FOR APPROPRIATE THRUST/DRAG COEFFICIENT                      FUNC0025
      IF(ETA.GE.A1) GO TO 10                                            FUNC0026
      IF(ETA.GE.A2) GO TO 11                                            FUNC0027
      AII=AI(3)                                                         FUNC0028
      BII=BI(3)                                                         FUNC0029
      CII=CI(3)                                                         FUNC0030
      GO TO 40                                                          FUNC0031
   11 AII=AI(2)                                                         FUNC0032
      BII=BI(2)                                                         FUNC0033
      CII=CI(2)                                                         FUNC0034
      GO TO 40                                                          FUNC0035
                                                                        FUNC0036
```

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10 AII=AI(1)                                                            FUNC0037
   BII=BI(1)                                                            FUNC0038
   CII=CI(1)                                                            FUNC0039
C  RIGHT-HAND-SIDE OF LINEAR D.E.'S FOR UDT,VDT,RDT AND PDT.            FUNC0040
C                                                                       FUNC0041
40 F1=(L*(XCOEFF(1)*(O**2)+XCOEFF(2)*(R**2)+XCOEFF(3)*R*P)+(XCOEFF(5    FUNC0042
  1)*V*R+XCOEFF(6)*W*Q)+((1/L)*(XCOEFF(7)*(U**2)+XCOEFF(8)*V**2+XCOEF   FUNC0043
  2F(9)*W**2+AII*U**2+BII*U*UO+CII*UO**2)+((1/L)*(U**2)*(XCOEFF(10)*    FUNC0044
  3(DELR**2)+XCOEFF(11)*(DELS**2)+XCOEFF(12)*(DELB**2))+((1/L)*(ETA-    FUNC0045
  41)*(XCOEFF(13)*(V**2)+XCOEFF(14)*(W**2)+XCOEFF(15)*(DELR*U)**2+XCO   FUNC0046
  5EFF(16)*(DELS*U)**2))-((2/(RHO*(L**3)))*(WT-B)*DSIN(THETAR))-(M*(W   FUNC0047
  6*Q-V*R-XG*(Q**2+R**2)+YG*P*Q+ZG*(P*R+O)))                           FUNC0048
C                                                                       FUNC0049
   Z=V                                                                  FUNC0050
   IF(Z.EQ.0.0D1) Z=0.1D-15                                             FUNC0051
F2=(L*(YCOEFF(3)*P*DABS(P)+YCOEFF(4)*P*Q+YCOEFF(5)*Q*R))+(YCOEFF(7      FUNC0052
  1)*Q*V+YCOEFF(8)*W*P+YCOEFF(9)*W*R+YCOEFF(10)*U*R+YCOEFF(11)*U*P+YC   FUNC0053
  2OEFF(12)*U*DABS(R)*DELR+YCOEFF(13)*U*DABS(Z)*DABS(VW)*DABS(R)+YC     FUNC0054
  3OEFF(19)*U*R*P*(ETA-1))+((1/L)*(YCOEFF(14)*(U**2)+YCOEFF(15)*U*V+YCO FUNC0055
  4EFF(16)*V*DABS(VW)+YCOEFF(17)*V*W+YCOEFF(18)*(U**2)*DELR))+((2/(RH   FUNC0056
  50*L**3))*(WT-B)*DCOS(THETAR)*DCOS(PHIR))+((1/L)*(ETA-1)*(YCOEFF(     FUNC0057
  620)*U*V+YCOEFF(21)*V*DABS(VW)+YCOEFF(22)*DELR*(U**2)))-(M*(U*R-W*P   FUNC0058
  7-YG*(R**2+P**2)+ZG*Q*R+XG*Q*P))                                     FUNC0059
C                                                                       FUNC0060
F3=(NCOEFF(3)*P*O+NCOEFF(4)*Q*R+NCOEFF(5)*R*DABS(R)+((1/L)*(NCOEFF     FUNC0061
  1(7)*W*R+NCOEFF(8)*W*P+NCOEFF(9)*V*Q+NCOEFF(10)*U*P+NCOEFF(11)*U*R+   FUNC0062
  2NCOEFF(12)*U*DABS(R)*DELR+NCOEFF(13)*DABS(VW)*R)+((1/L**2)*(NCOEF    FUNC0063
  3F(14)*U**2+NCOEFF(15)*U*V+NCOEFF(16)*V*DABS(VW)+NCOEFF(17)*W*V+NCC   FUNC0064
  4EFF(18)*(U**2)*DELR))+((2/RHO*L**5))*((XG*WT-XB*B)*DCCS(THETAR)*DS   FUNC0065
  5IN(PHIR))+(YG*WT-YB*B)*DSIN(THETAR)))+((1/L)*(ETA-1)*NCOEFF(19)*U*R  FUNC0066
  6)+((1/L**2)*(ETA-1)*(NCOEFF(20)*U*V+NCOEFF(21)*V*DABS(VW)+NCOEFF(2   FUNC0067
  72)*DELR*U**2))-((IX-IY)*Q*R)+(IYZ*(P*R+QDT))-(IXY*(Q**2-P**2))-(IX   FUNC0068
  8Z*P*R)-(M*(1/L**2)*(XG*(U*R-W*P)-YG*(W*Q-V*R)))                     FUNC0069
C                                                                       FUNC0070
F4=(KCOEFF(3)*Q*R+KCOEFF(4)*P*Q+KCOEFF(5)*P*DABS(P))+((1/L)*(KCOEF     FUNC0071
  1F(6)*U*P+KCOEFF(7)*U*R+KCOEFF(9)*V*Q+KCOEFF(10)*W*R+KCOEFF(11)*W*R   FUNC0072
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2))+((1/L**2)*(KCOEFF(12)*(U**2)+KCOEFF(13)*V*U+KCOEFF(14)*V*DABS(V    FUNC0073
3W)+KCOEFF(15)*V*W+KCOEFF(16)*(U**2)*DELR+KCOEFF(17)*(U**2)*(ETA-1)    FUNC0074
4))+((2/(RHO*L**5))*((YG*WT-YB*B)*DCOS(THETAR)*DCOS(PHIR)-(ZG*WT-ZB    FUNC0075
5*B)*DCOS(THETAR)*DSIN(PHIR))-((IZ-IY)*Q*R)+(IXZ*Q*P)-((R**2-Q**2)     FUNC0076
6*IYZ)-IXY*(P*R-QDT)-((M*(1/L**2))*(YG*(V*P-U*Q)-ZG*(U*R-W*P)))        FUNC0077
                                                                      FUNC0078
      RETURN                                                          FUNC0079
      END                                                             FUNC0080
C
```

```
C
      SUBROUTINE RUDDER
C
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)                                   RUD10001
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI( RUD10002
     13),BI(3),CI(3),XCOEFF(16)                                         RUD10003
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ                                        RUD10004
      COMMON   T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG                    RUD10005
      COMMON    XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG, RUD10006
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,DO,DELS,DELB,THETA,PH RUD10007
     2I,A1,A2                                                           RUD10008
      COMMON F1,F2,F3,F4,WT,B                                            RUD10009
      COMMON DELR,RHO,TR,RUD1,U3                                         RUD10010
      COMMON I1,I2                                                       RUD10011
C                                                                        RUD10012
C     TEST FOR WHERE IN THE TURN THE VEHICLE IS AND CHANGE THE AMOUNT OF RUD10013
C     RUDDER AS APPROPRIATE.                                            RUD10014
C                                                                        RUD10015
      R1=R                                                              RUD10016
      T22=T2                                                            RUD10017
      IF(R.EQ.0.0)  R1=1.0D-20                                          RUD10018
      APSI=DABS(PSI)                                                    RUD10019
      AR1=DABS(R1)                                                      RUD10020
      TT=(COURSE-APSI)/AR1                                              RUD10021
      T3=T2*2.0                                                         RUD10022
      DELR1=DABS(DELR)                                                  RUD10023
      IF(I2.GT.0)  GC TO 3                                              RUD10024
      IF(I1.GT.0)  GC TO 2                                              RUD10025
      IF(TT.LE.T3)  GO TO 2                                             RUD10026
      IF(TR.GT.0.0D+1)  T22=TR                                          RUD10027
      IF(T.LT.T22)  GO TO 1                                             RUD10028
      DELR1=DABS(RUDAMT)                                                RUD10029
      GO TO 5                                                           RUD10030
    1 DELR1=DELR1+DELT*RRATE                                            RUD10031
    5 DELR=DSIGN(DELR1,RUDAMT)                                          RUD10032
      RETURN                                                           RUD10033
```

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```
    2 I1=1                                              RUD10037
      DELR1=DELR1-DELT*RRATE                            RUD10038
      IF(DELR1.LE.0.0) GO TO 3                          RUD10039
      GO TO 5                                           RUD10040
    3 I2=1                                              RUD10041
      DELR=0.0D+1                                       RUD10042
      RETURN                                            RUD10043
      END                                               RUD10044
```

```
C      SUBROUTINE ALTRUD                                              RUD20001
                                                                      RUD20002
C                                                                     RUD20003
       IMPLICIT REAL*8(A-H),REAL*8(J-Z)                               RUD20004
       REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI( RUD20005
      13),BI(3),CI(3),XCOEFF(16)                                      RUD20006
       REAL*8 IX,IY,IZ,IXY,IXZ,IYZ                                    RUD20007
       COMMON  T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG                 RUD20008
       COMMON   XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG, RUD20009
      1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,DELB,THETA,PH RUD20010
      2I,A1,A2                                                        RUD20011
       COMMON F1,F2,F3,F4,WT,B                                        RUD20012
       COMMON DELR,RHO,TR,RUD1,U3                                     RUD20013
       COMMON I1,I2                                                   RUD20014
                                                                      RUD20015
C                                                                     RUD20016
C      THIS SUBROUTINE MAINTAINS THE RUDDER AMOUNT AT LESS THAN       RUD20017
C      MAXIMUM ORDERED RUDDER UNTIL U IS LESS THAN 1/3 UO. ALTRUD     RUD20018
C      SHOULD NOT BE USED FOR COURSE CHANGES OF LESS THAN ABOUT 90 DEGS. RUD20019
                                                                      RUD20020
       R1=R                                                           RUD20021
       IF(R.EQ.0.0) R1=1.0D-20                                        RUD20022
       T23=(DABS(RUD1)/RRATE)+TLAG                                    RUD20023
       APSI=DABS(PSI)                                                 RUD20024
       AR1=DABS(R1)                                                   RUD20025
       TT=(COURSE-APSI)/AR1                                           RUD20026
       T3=T2*2.0                                                      RUD20027
       DELR1=DABS(DELR)                                               RUD20028
       TR=T2+T-T23                                                    RUD20029
       IF(I2.GT.0)  GO TO 3                                           RUD20030
       IF(I1.GT.0)  GO TO 2                                           RUD20031
       IF(TT.LE.T3) GO TO 2                                           RUD20032
       IF(T.LT.T23) GO TO 1                                           RUD20033
       DELR1=DABS(RUD1)                                               RUD20034
       GO TO 5                                                        RUD20035
     1 DELR1=DELR1+DELT*RRATE                                         RUD20036
     5 DELR=DSIGN(DELR1,RUDAMT)
```

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```
      RETURN                                    RUD20037
    2 I1=1                                       RUD20038
      DELR1=DELR1-DELT*RRATE                     RUD20039
      IF(DELR1.LE.0.0) GO TO 3                   RUD20040
      GO TO 5                                    RUD20041
    3 I2=1                                       RUD20042
      DELR=0.0D+1                                RUD20043
      RETURN                                     RUD20044
      END                                        RUD20045
```

A.3 SAMPLE OUTPUT

```
•••••••••••••••••••••••••••••••••••••
•••  OUTPUT FOR SUBMARINE 2 - NO MODS
•••••••••••••••••••••••••••••••••••••
```

UO =0.230D+02   RUDDER RATE =0.400D+01   COURSE CHANGE =0.360D+03

AMOUNT OF RUDDER =0.350D+02   DELTA T =0.500D+00

T (SEC)	U (KTS)	V (KTS)	R (DEG/SEC)	PHI (DEG)	UDT (KTS/HRS)	VDT (KTS/HRS)	RDT (DEG/SEC/SEC)	PDT (DEG/SEC/SEC)	UBLR (DEG)	CRS (DEG)	X (YDS)	Y (YDS)
0.0	0.230D+02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150D+01	0.230D+02	0.0	0.0	0.0	-.822D-03	0.344D-01	-.876D-01	0.133D+00	0.200D+01	0.0	0.647D+01	0.0
0.200D+01	0.230D+02	0.172D-01	-.438D-01	0.166D-01	-.331D-02	0.718D-01	-.172D+00	0.220D+00	0.400D+01	-.109D-01	0.129D+02	-.608D-02
0.250D+01	0.230D+02	0.531D-01	-.130D+00	0.774D-01	-.762D-02	0.112D+00	-.252D+00	0.265D+00	0.600D+01	-.543D-01	0.194D+02	-.272D-01
0.300D+01	0.230D+02	0.109D+00	-.255D+00	0.199D+00	-.141D-01	0.154D+00	-.327D+00	0.273D+00	0.800D+01	-.151D+00	0.259D+02	-.749D-01
0.350D+01	0.230D+02	0.186D+00	-.419D+00	0.387D+00	-.231D-01	0.199D+00	-.398D+00	0.250D+00	0.100D+02	-.319D+00	0.324D+02	-.163D+00
0.400D+01	0.230D+02	0.286D+00	-.618D+00	0.641D+00	-.352D-01	0.245D+00	-.463D+00	0.200D+00	0.120D+02	-.578D+00	0.388D+02	-.309D+00
0.450D+01	0.230D+02	0.409D+00	-.849D+00	0.952D+00	-.513D-01	0.293D+00	-.522D+00	0.127D+00	0.140D+02	-.945D+00	0.453D+02	-.531D+00
0.500D+01	0.229D+02	0.555D+00	-.111D+01	0.130D+01	-.719D-01	0.340D+00	-.575D+00	0.357D-01	0.160D+02	-.143D+01	0.517D+02	-.848D+00
0.550D+01	0.229D+02	0.725D+00	-.140D+01	0.167D+01	-.980D-01	0.386D+00	-.620D+00	-.707D-01	0.180D+02	-.196D+01	0.582D+02	-.128D+01
0.600D+01	0.228D+02	0.918D+00	-.171D+01	0.204D+01	-.130D+00	0.430D+00	-.658D+00	-.188D+00	0.200D+02	-.284D+01	0.646D+02	-.186D+01
0.650D+01	0.228D+02	0.113D+01	-.204D+01	0.236D+01	-.170D+00	0.472D+00	-.687D+00	-.314D+00	0.220D+02	-.377D+01	0.710D+02	-.260D+01
0.700D+01	0.227D+02	0.137D+01	-.238D+01	0.265D+01	-.216D+00	0.508D+00	-.706D+00	-.445D+00	0.240D+02	-.488D+01	0.773D+02	-.353D+01
0.750D+01	0.226D+02	0.162D+01	-.273D+01	0.283D+01	-.271D+00	0.540D+00	-.715D+00	-.577D+00	0.260D+02	-.616D+01	0.836D+02	-.466D+01
0.800D+01	0.225D+02	0.189D+01	-.309D+01	0.287D+01	-.333D+00	0.565D+00	-.713D+00	-.706D+00	0.280D+02	-.761D+01	0.898D+02	-.603D+01
0.850D+01	0.223D+02	0.218D+01	-.345D+01	0.276D+01	-.402D+00	0.582D+00	-.700D+00	-.825D+00	0.300D+02	-.925D+01	0.959D+02	-.764D+01
0.900D+01	0.221D+02	0.247D+01	-.380D+01	0.246D+01	-.516D+00	0.638D+00	-.797D+00	-.745D+00	0.350D+02	-.111D+02	0.102D+03	-.952D+01
0.950D+01	0.218D+02	0.279D+01	-.420D+01	0.196D+01	-.579D+00	0.605D+00	-.669D+00	-.105D+01	0.350D+02	-.131D+02	0.108D+03	-.117D+02
0.100D+02	0.215D+02	0.309D+01	-.453D+01	0.124D+01	-.641D+00	0.565D+00	-.580D+00	-.124D+01	0.350D+02	-.152D+02	0.113D+03	-.141D+02

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A.4 INPUT/OUTPUT VARIABLES

VARIABLE	MEANING	FORMAT	DIMENSIONS	INPUT/OUTPUT
AI, BI, CI	Set of constants representing the propellar thrust in the X-equation	D10.4	ND	I
A1, A2	Limits used for selecting the proper propeller thrust	D10.4	ND	I
B	Ship's buoyancy	D10.4	lbs	I
COURSE	Amount of ordered course change	D10.4	deg	I, 0
DELB	Initial sail-plane deflection	D10.4	deg	I
DELGM	Amount the original GM is to be changed	D10.4	ft	I
DELRO	Rudder deflection at time T	D9.3	deg	0

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[1] ND = non-dimensional

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DELS	Initial sternplane deflection	D10.4	deg	I
DELT	Time increment used in iteration	D10.4	sec	I, 0
ID	40 character alpha-numeric heading	A1	ND	I, 0
INDEX	If >0, read new submarine coefficients. If ≤0, run same sub-marine for new initial conditions.	I2	ND	I
IX, IY, IZ IXY, IXZ, IYZ	Moments of inertia	D10.4	ND	I
KCOEFF	K-equation coefficients	D10.4	ND	I
L	Ship's over-all length	D10.4	ft	I
M	Ship's mass	D10.4	ND	I
NCOEFF	N-equation coefficients	D10.4	ND	I

P, Q, R	Initial angular velocity about the x, y and z axes, respectively	D10.4	deg/sec	I
PDT, QDT, RDT	Initial angular accelerations about the x, y and z axes, respectively	D10.4	deg/sec^2	I
PDTO, RDTO	Angular accelerations about the x and z axes, respectively	D9.3	deg/sec^2	O
PHI	Initial angle of roll	D10.4	deg	I
PHIO	Roll angle at time T	D9.3	deg	O
PSI	Initial angle of yaw	D10.4	deg	I
PSIO	Yaw angle at time T	D9.3	deg	O
RHO	Sea water density	D10.4	slugs/ft^3	I
RO	Angular velocity about the z axis at time T	D9.3	deg/sec	O

RRATE	Average rudder rate	D10.4	deg/sec	I, O
RUDAMT	Maximum ordered rudder deflection	D10.4	deg	I,O
RUD1	Value of reduced rudder deflection associated with sub-routine ALTRUD	D10.4	deg	I, O
T	Present time	D9.3	sec	O
THETA	Initial angle of pitch	D10.4	deg	I
TLAG	Time lag of of the control system	D10.4	sec	I
U, V, W	Initial forward, lateral and vertical velocities	D10.4	kts	I
UDT, VDT, WDT	Initial forward, lateral and vertical accelerations	D10.4	kts/hr	I

UDTO, VDTO	Forward and lateral acceleration, respectively at time T	D9.3	kts/hr	O
UO	Initial forward velocity	D9.3	kts	O
UOO, VO	Forward and lateral velocities, respectively at time T	D9.3	kts	O
WT	Ship's weight	D10.4	lbs	I
XB, YB, ZB	The x, y, z position of the center of buoyancy	D10.4	ft	I
XCOEFF	X-equation coefficients	D10.4	ND	I
XG, YG, ZG	The x, y, z position of the center of gravity	D10.4	ft	I
XX, YY	The moving x and y coordinates of the submarine's origin with respect to a fixed coordinate system, whose origin is the point where the turn begins	D9.3	yds	O

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YCOEFF   Y-equation   D10.4   ND   I
                coefficients

B.1 AXIAL FORCE

$$m\left[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2} \ell^4 \left[X_{qq}'q^2 + X_{rr}'r^2 + X_{rp}'rp\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}'\dot{u} + X_{vr}'vr + X_{wq}'wq\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{uu}'u^2 + X_{vv}'v^2 + X_{ww}'w^2\right]$$

$$+ \frac{\rho}{2} \ell^2 u^2 \left[X_{\delta_r \delta_r}'\delta_r^2 + X_{\delta_s \delta_s}'\delta_s^2 - X_{\delta_b \delta_b}'\delta_b^2\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[a_i u^2 + b_i u u_c + c_i u_c^2\right]$$

$$- (W - B) \sin \theta$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{vv\eta}'v^2 + X_{ww\eta}'w^2 + X_{\delta_r \delta_r \eta}'\delta_r^2 u^2\right.$$

$$\left. + X_{\delta_s \delta_s}'\delta_s^2 u^2\right](\eta - 1)$$

B.2 LATERAL FORCE

$$m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$

$$+ \frac{\rho}{2}\ell^4\left[Y_{\dot{r}}'\dot{r} + Y_{\dot{p}}'\dot{p} + Y_{p|p|}'p|p| + Y_{pq}'pq + Y_{qr}'qr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Y_{\dot{v}}'\dot{v} + Y_{vq}'vq + Y_{wp}'wp + Y_{wr}'wr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Y_r'ur + Y_p'up + Y_{|r|\delta r}'u|r|\delta_r\right.$$
$$\left. + Y_{v|r|}'\frac{v}{|v|}|(v^2 - w^2)^{\frac{1}{2}}||r|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_*'u^2 + Y_v'uv + Y_{v|v|}'v|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_{vw}'vw + Y_{\delta r}'u^2\delta_r\right]$$

$$+ (W - B)\cos\theta\sin\varphi$$

$$+ \frac{\rho}{2}\ell^3 Y_{r\eta}'ur(\eta - 1)$$

$$+ \frac{\rho}{2}\ell^2\left[Y_{v\eta}'uv + Y_{v|v|\eta}'v|(v^2 + w^2)^{\frac{1}{2}}| + Y_{\delta r\eta}'\delta_r u^2\right](\eta - 1)$$

B.3 ROLLING MOMENT

$$I_x \dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - q)I_{xy}$$

$$+ m\left[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\right] =$$

$$+ \frac{\rho}{2}\ell^5\left[K_{\dot{p}}'\dot{p} + K_{\dot{r}}'\dot{r} + K_{qr}'qr + K_{pq}'pq + K_{p|p|}'p|p|\right]$$

$$+ \frac{\rho}{2}\ell^4\left[K_p'up + K_r'ur + K_{\dot{v}}'\dot{v}\right]$$

$$+ \frac{\rho}{2}\ell^4\left[K_{vq}'vq + K_{wp}'wp + K_{wr}'wr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[K_*'u^2 + K_v'uv + K_{v|v|}'v|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+ \frac{\rho}{2}\ell^3\left[K_{vw}'vw + K_{\delta r}'u^2\delta r\right]$$

$$+ (y_G W - y_B B)\cos\theta\cos\varphi - (z_G W - z_B B)\cos\theta\sin\varphi$$

$$+ \frac{\rho}{2}\ell^3 K_{*\eta}'u^2(\eta - 1)$$

B.4 YAWING MOMENT

$$I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx}$$

$$+ m\left[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\right] =$$

$$+ \frac{\rho}{2}\ell^5\left[N_{\dot{r}}{}'\dot{r} + N_{\dot{p}}{}'\dot{p} + N_{pq}{}'pq + N_{qr}{}'qr + N_{r|r|}{}'r|r|\right]$$

$$+ \frac{\rho}{2}\ell^4\left[N_{\dot{v}}{}'\dot{v} + N_{wr}{}'wr + N_{wp}{}'wp + N_{vq}{}'vq\right]$$

$$+ \frac{\rho}{2}\ell^4\left[N_p{}'up + N_r{}'ur + N_{|r|\delta r}{}'u|r|\delta r + N_{|v|r}{}' |(v^2 + w^2)^{\frac{1}{2}}|r|\right]$$

$$+ \frac{\rho}{2}\ell^3\left[N_*{}'u^2 + N_v{}'uv + N_{v|v|}{}'v|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+ \frac{\rho}{2}\ell^3\left[N_{vw}{}'vw + N_{\delta r}{}'u^2\delta r\right]$$

$$+ (x_G W - x_B B)\cos\theta\sin\varphi + (y_G W - y_B B)\sin\theta$$

$$+ \frac{\rho}{2}\ell^4 N_{r\eta}{}'ur\,(\eta - 1)$$

$$+ \frac{\rho}{2}\ell^3\left[N_{v\eta}{}'uv + N_{v|v|\eta}{}'v|(v^2 + w^2)^{\frac{1}{2}}| + N_{\delta r\eta}{}'\delta_r u^2\right](\eta - 1)$$